AMPLITUDES FOR CHARGE AND HYPERCHARGE EXCHANGE REACTIONS

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The measurement of the $\pi^- p \to \pi^0 n$ polarization [1] finally destroyed any lingering hope that the conventional absorptive modifications [2],[3] to Regge poles would represent the data in the intermediate energy region ($\sim$6 GeV/c). Instead the data (or rather the πN amplitudes extracted directly from the data) have been used to suggest detailed [4] or ad hoc [5] corrections to the absorptive prescriptions. This has reduced the absorption model to an empirical basis from which it looses much of its predictive power.

A first step in the understanding of the corrections to the Regge pole contributions is to extract, wherever possible, the amplitudes in a model independent way from the experimental data. In terms of s channel helicity amplitudes a complete set of experimental observables for spin 0 - spin $\frac{1}{2}$ scattering are

$$\frac{d\sigma}{dt} = |F_{++}|^2 + |F_{+-}|^2$$

$$P\frac{d\sigma}{dt} = 2\,\mathrm{Im}\left(F_{++}F_{+-}^*\right)$$

$$R \frac{d\sigma}{dt} = 2\,\mathrm{Re}\left(F_{++}F_{+-}^{*}\right)\sin\theta_T - \left(|F_{++}|^2 - |F_{+-}|^2\right)\cos\theta_T$$

$$A \frac{d\sigma}{dt} = \left(|F_{++}|^2 - |F_{+-}|^2\right)\sin\theta_T + 2\,\mathrm{Re}\left(F_{++}F_{+-}^{*}\right)\cos\theta_T \qquad (1)$$

where R and A are the Wolfenstein spin-correlation parameters ($P^2 + R^2 + A^2 = 1$) and θ_T is the scattering angle of the target in the helicity rest frame of the recoil particle ; at high energies and small t, $\theta_T \approx \pi/2$. Experimentally, R and A are the components of the recoil baryon polarization for scattering from a polarized target. Given such a set of observables it is possible to solve Eqs. (1) and determine F_{++} and F_{+-} up to an over-all phase.

1. - πN AMPLITUDE ANALYSIS

The recent measurements of R^- and P^0 (where superscripts ±, 0 refer to $\pi^{\pm}p \to \pi^{\pm}p$, $\pi^-p \to \pi^0 n$ respectively) made an amplitude analysis possible for πN scattering at 6 GeV/c. Since there are two isospin states (I = 0,1 in the t channel) we have four (complex) amplitudes and seven parameters at each t value. Using the new data together with the existing $d\sigma^{\pm,0}/dt$, $P^{\pm}$ measurements, Halzen and Michael [6)] solved for the moduli and relative phases of the s channel helicity amplitudes in the range $0 \leq -t \leq 0.625$ GeV^2. The ambiguities are resolved by requiring continuity of the solution in t, from the expected t = 0 behaviour. The resulting amplitudes are consistent with an earlier Barger and Phillips [7)] finite energy sum rule (FESR) and Regge analysis of the then existing polarization and cross-section data. This was to be expected since the high energy amplitudes of Ref. 7) had sufficient freedom of parametrization to give a good fit to the data and since the analysis correctly predicted the form of the new R^- and P^0 data. Moreover the Barger and Phillips amplitudes have the absolute phase determined from πN phase shifts through the use of fixed t FESR's. We compare the ρ exchange amplitudes resulting from these two analyses in Fig. 1. We see that the ρ flip $(n=1)$ amplitude, and also $\mathrm{Re}\, F_{++}(t)$, have the Regge pole like structure $(-t)^{n/2}(1 - e^{-i\pi\alpha(t)})e^{bt}$. The partial wave decompositions of the ρ amplitudes are shown in Fig. 2. We see that the imaginary parts of F^J are peripheral with an interaction radius of $b \approx 1$ Fermi, in agreement with the general rule proposed by Harari [9]. (A similar behaviour is found [10] for the imaginary part of the ω exchange non-flip amplitude by considering the difference of the $K^{\pm}p$ cross-sections, suitably normalized.) The real parts of F^J do not have a peripheral behaviour. In absorption language we require a complicated prescription which strongly or over absorbs the low partial waves in $\mathrm{Im}\, F^J_{++}$, and essentially no absorption for $\mathrm{Re}\, F^J_{++}$ and F^J_{+-} so that they retain their Regge pole like behaviour [cf. Fig. 3 of Ref. 11]. However, there is a significant high J tail $(J > 12)$ in $\mathrm{Im}\, F^J_{++}$ which cannot easily be explained in the absorption approach [11].

2. - AMPLITUDES FOR HYPERCHARGE EXCHANGE REACTIONS

The line reversed hypercharge exchange reactions $(\pi N \to KY$ and $\bar{K}N \to \pi Y)$ are of particular importance for obtaining insight into the form of the corrections to Regge pole exchanges. One reason is that in Λ and Σ^+ production processes the weak decay of the hyperon analyzes its polarization and so a complete set of observables can be obtained using a polarized target but without having to perform the double scattering determination of the recoil polarization. Secondly these processes are probably the best source of information on tensor exchange (K^{**}) amplitudes as well as providing additional information on vector exchange (K^*). Finally, the relative importance of the s channel non-flip amplitudes as compared to the ρ, A_2 exchange reactions (see the last column of the Table) suggest that hypercharge exchange reactions are a good place to study the modifications to Regge pole exchanges.

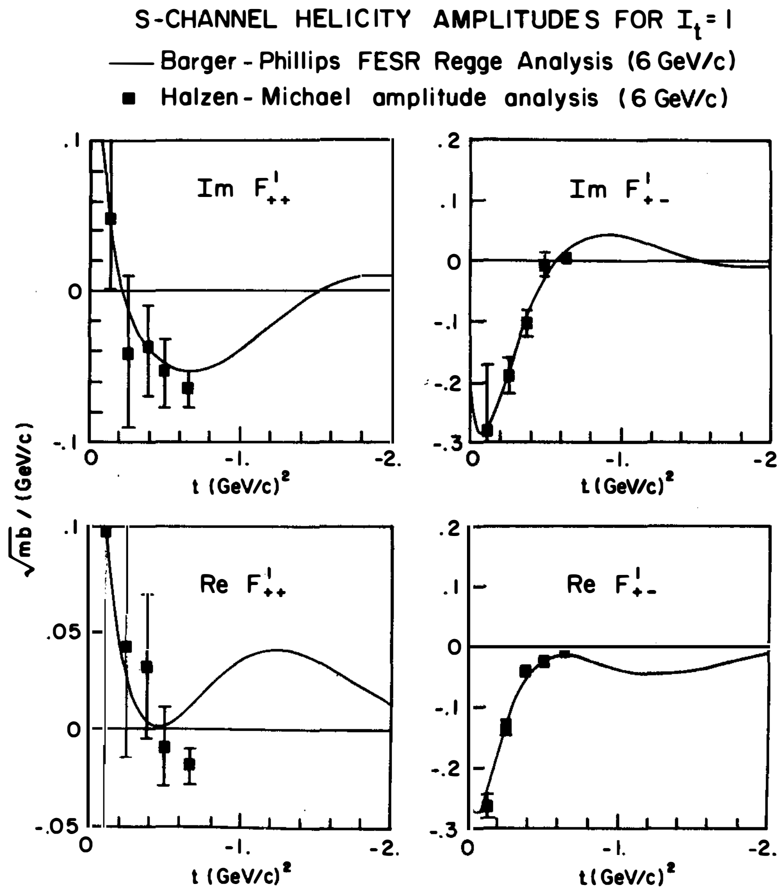

Figure 1 The ρ exchange amplitudes for πN scattering at 6 GeV/c taken from Ref. 8). The over-all phase found in the Barger and Phillips analysis has been used to rotate the Halzen and Michael solution. At $t = 0$, $\mathrm{Im}\, F^1_{++} = 0.281 \pm 0.032$ and $\mathrm{Re}\, F^1_{++} = 0.312 \pm 0.042$ $\sqrt{mb}/(GeV/c)$.

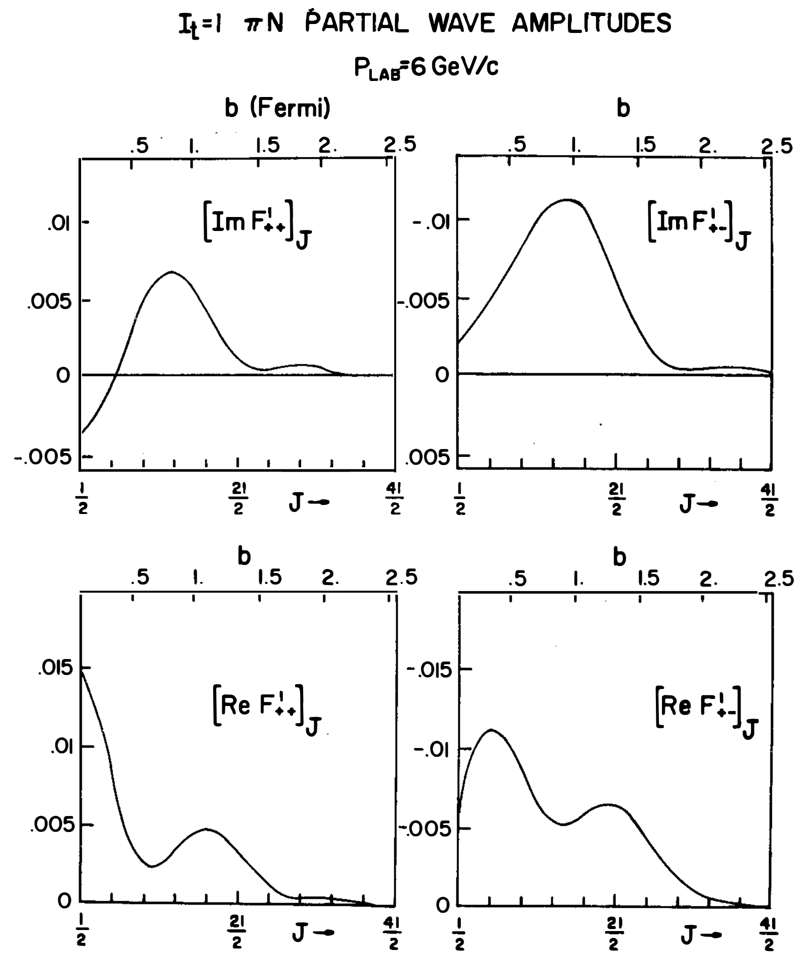

Figure 2 The partial wave decomposition of the ρ exchange amplitudes taken from Ref. 8).

Process	Exchange amplitudes	$\lvert F_{+-}\rvert / \lvert F_{++}\rvert$ $(0.1 < -t < 0.35)$
$\pi^- p \to \pi^o n$	$-\sqrt{2}\ \rho$	~ 3
$\pi^- p \to \eta_8 n$	$\sqrt{2/3}\ A_2$	
$K^- p \to \bar{K}^o n$	$-(A_2 + \rho)$	
$K^+ n \to K^o p$	$(A_2 - \rho)$	
$\pi^- p \to K^o \Lambda$	$-\sqrt{1/6}(2F + 1)(K^{**} + K^*)$	$\sim 9/8$
$K^- p \to \pi^o \Lambda$	$\sqrt{1/12}(2F + 1)(K^{**} - K^*)$	
$\pi^+ p \to K^+ \Sigma^+$	$-(2F - 1)(K^{**} + K^*)$	$\sim 3/4$
$K^- p \to \pi^- \Sigma^+$	$-(2F - 1)(K^{**} - K^*)$	

<u>TABLE</u>

The composition of the amplitudes assuming SU(3) octet exchange with equal F/D ratios for the vector (ρ or K^*) and tensor (A_2 or K^{**}) exchanges. The charge and hypercharge exchange amplitudes differ by an over-all SU(3) breaking factor arising from the difference of the $\rho - A_2$ and $K^* - K^{**}$ trajectories [12]. F is the fraction of F type coupling at the baryon vertex defined so that $F + D = 1$. The $\pi^- p \to \pi^o n$ s channel flip to non-flip ratio is obtained from the Halzen and Michael [6] amplitude analysis, whereas the values for the hypercharge exchange reactions are estimated using flip and non-flip F/F+D ratios of 1/4 and 3/2 respectively.

First let us recall the duality diagram predictions of the exchange degenerate (EXD) $K^{*}-K^{**}$ pole model for these reactions. At a given energy the amplitudes have the form

$$T_{-} \equiv T(\bar{K}N \to \pi\Sigma) = K^{**} - K^{*} = -\beta(t)$$
$$T_{+} \equiv T(\pi N \to K\Sigma) = K^{**} + K^{*} = -\beta(t)\, e^{-i\pi\alpha(t)} \tag{2}$$

and similarly for the Λ processes. The first reaction is described by a real amplitude while the line-reversed reaction has an amplitude with a rotating phase determined by the EXD trajectory. This model thus leads to equal line-reversed cross-sections and zero polarizations

$$\frac{d\sigma_{+}}{dt} = \frac{d\sigma_{-}}{dt}, \qquad P_{+} = P_{-} = 0. \tag{3}$$

However, the data over a range of energies show

$$\frac{d\sigma_{-}}{dt} > \frac{d\sigma_{+}}{dt} \tag{4}$$

by up to a factor of 2 and large polarizations. The data in the region of 4 GeV/c are shown in Figs. 3 and 4. Although the conventional absorptive corrections [13] to the s channel non-flip amplitudes using a Pomeron with slope $\alpha_{P}' \sim 0.5$ reproduce all the features of the polarizations they give

$$\frac{d\sigma_{-}}{dt} < \frac{d\sigma_{+}}{dt} \tag{5}$$

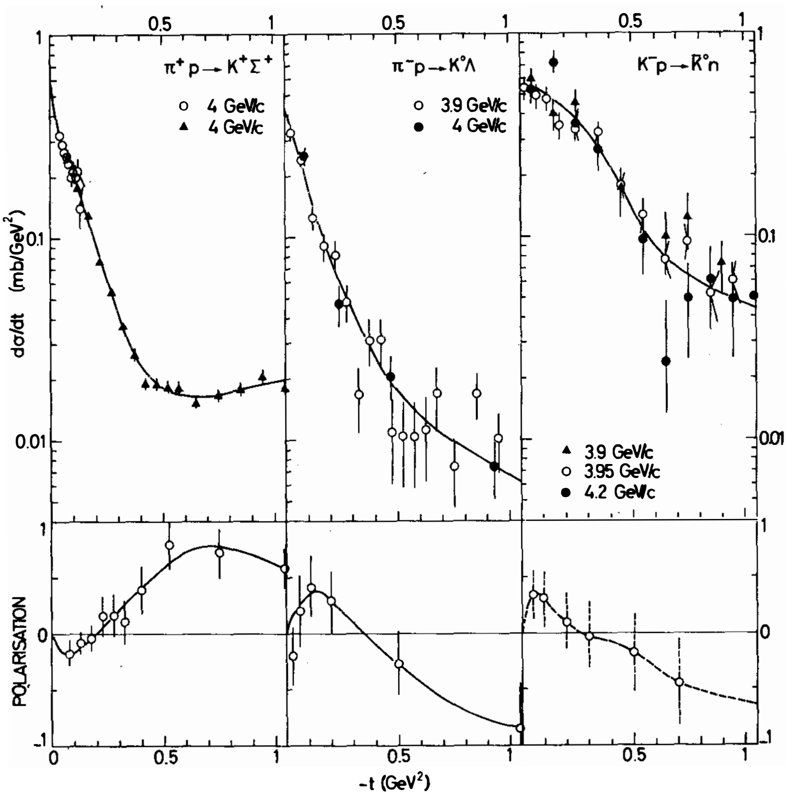

Figure 3 The cross-section and polarization data near 4 GeV/c together with interpolating curves for reactions described by "rotating phase" pole amplitudes, taken from Ref. 12). The dashed curve for $P(K^-p \to K^0n)$ is a prediction. The ratio of the flip to non-flip amplitudes is expected to increase from the left to the right reaction.

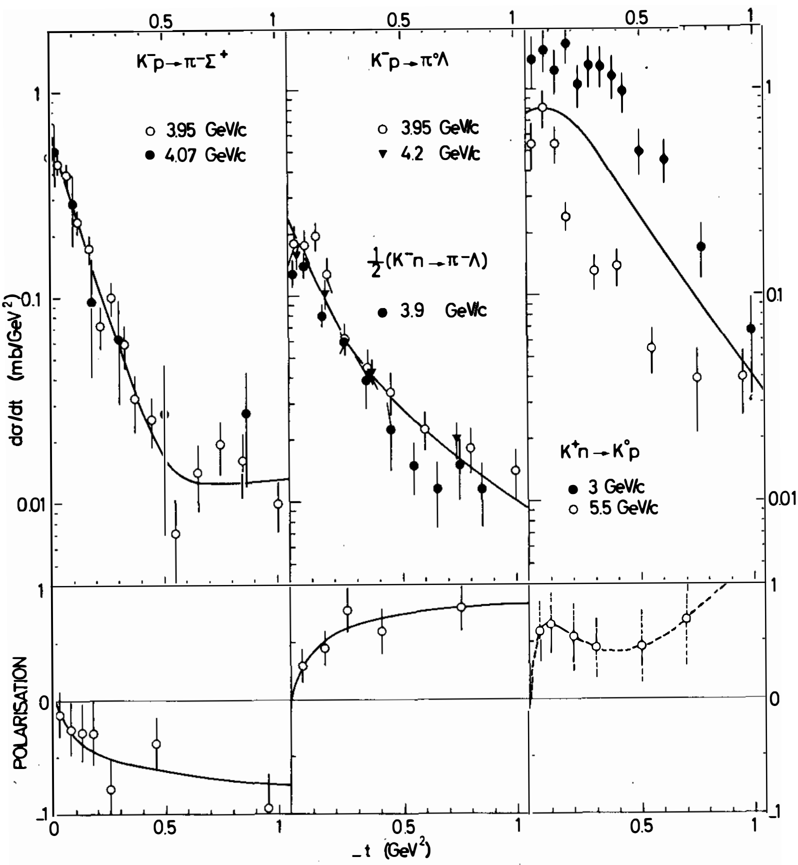

Figure 4 The same as Fig. 3 but for processes with "real" amplitudes.

in contradiction with the data. The result of Eq. (5) can be anticipated since the pole amplitude of Eq. (2) that oscillates in t is absorbed less by a structureless Pomeron and so the resultant amplitude is larger.

To determine the form of the corrections to the pole amplitudes for the hypercharge exchange data, model independent deductions of the amplitudes from experiment are needed, such as those we described for $\pi^- p \to \pi^0 n$. However, R and A measurements are not yet available and so some theoretical input is required for an amplitude analysis. We make the plausible assumption that the s channel helicity flip amplitudes, T^f, are essentially pole-like

$$\begin{aligned} T_-^f &= -\rho_f(t) \\ T_+^f &= -\rho_f(t)\, e^{-i\pi\alpha(t)} . \end{aligned} \tag{5}$$

This assumption is motivated by the results of the πN amplitude analysis and by the success of the Barger-Cline [14] SU(3) sum rule for the flip dominated charge exchange processes [15]. No assumption is made for the s channel non-flip amplitudes, T^{nf}. To facilitate comparison with the EXD Regge forms we parametrize them as

$$\begin{aligned} T_-^{nf} &= -\rho_-\, e^{i\phi_-} \\ T_+^{nf} &= -\rho_+\, e^{i(\phi_+ - \pi\alpha)} \end{aligned} \tag{7}$$

where the magnitudes, $\rho_\pm(t)$, and the (EXD breaking) phases $\phi_\pm(t)$ are to be determined directly from the cross-section and polarization data for the line reversed reactions

$$\frac{d\sigma_\pm}{dt} = |\rho_\pm|^2 + |\rho_f|^2$$
$$P_\pm \frac{d\sigma_\pm}{dt} = 2\rho_f \rho_\pm \sin\phi_\pm . \tag{8}$$

This can be accomplished given the modulus *) of the flip amplitude $|\rho_f|$. At present the highest energy where a complete set of polarization and cross-section data exist is 4 GeV/c (see Figs. 3 and 4).

Fortunately the data impose fairly restrictive bounds on $|\rho_f(t)|$. Using Eqs. (8) we see that the bounds $|\sin\phi| \leq 1$ may, on eliminating $\rho_\pm$, be expressed in the form

$$\frac{1}{2}\frac{d\sigma_\pm}{dt}\left(1-\sqrt{1-P_\pm^2}\right) \leq |\rho_f|^2 \leq \frac{1}{2}\frac{d\sigma_\pm}{dt}\left(1+\sqrt{1-P_\pm^2}\right). \tag{9}$$

The larger $|P|$ the tighter the bounds, moreover $|\rho_f|$ is constrained by the data from two reactions and the bounds are improved as the line reversal inequality of the cross-sections increases. The bounds, which are particularly restrictive for $|\rho_f^\Lambda|$, are shown in Ref. 16), together with the values used for $|\rho_f^\Sigma|$.

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*) There is a slight difference between the definitions of the flip and non-flip ρ's. $\rho_\pm$ are moduli as defined, whereas ρ_f, though real, can have a sign. Our convention is to take ρ_f positive in the Σ reactions which means ρ_f is negative for Λ reactions.

Using these forms of ρ_f and the data interpolations shown in Figs 3 and 4, we solve Eqs. (8) for $\rho_\pm$ and $\phi_\pm$ and express the results in terms of the K^* and K^{**} non-flip contributions

$$\left.\begin{matrix} K^* \\ K^{**} \end{matrix}\right\} = -\frac{1}{2}\left[\rho_+ e^{i(\phi_+ - \pi\alpha)} \mp \rho_- e^{i\phi_-}\right]. \tag{10}$$

There are four sets of solutions corresponding to the phase ambiguities and we select the solution with $\mathrm{Re}\, K^*/\mathrm{Im}\, K^* > 0$ and $\mathrm{Re}\, K^{**}/\mathrm{Im}\, K^{**} < 0$ at $t = 0$ to be in agreement with the signs of the pole amplitudes. The resulting non-flip contributions are shown in Fig. 5 based on a trajectory $\alpha = 0.4 + t$. The qualitative structure of the results is independent of the particular choice of the flip contributions providing $|\rho_f|^2$ is taken smooth in t and within the bounds given by Eqs. (9). The K^* non-flip amplitude has a similar structure to the ρ non-flip amplitude (cf. Fig. 1). That is in absorption language $\mathrm{Im}\, K^*_{++}$ is strongly absorbed whereas $\mathrm{Re}\, K^*_{++}$ suffers little or no absorption. On the other hand looking at tensor exchange we see that $\mathrm{Im}\, K^{**}_{++}$ does not have the "cross-over" zero at $t \approx -0.2$ and so is evidence against Harari's general rule [9].

Confirmation of the form of the amplitudes comes from $K^- p \to \eta_8 \Lambda$ which involves K^* and K^{**} exchanges in the combination $K^{**}_\Lambda + 3K^*_\Lambda$. For the non-flip amplitudes of Fig. 5, we observe an almost complete zero of both the real and imaginary parts of this combination for $t \approx -0.3$ in agreement with the striking dip seen [17] in the $K^- p \to \eta\Lambda$ cross-section at 4 GeV/c.

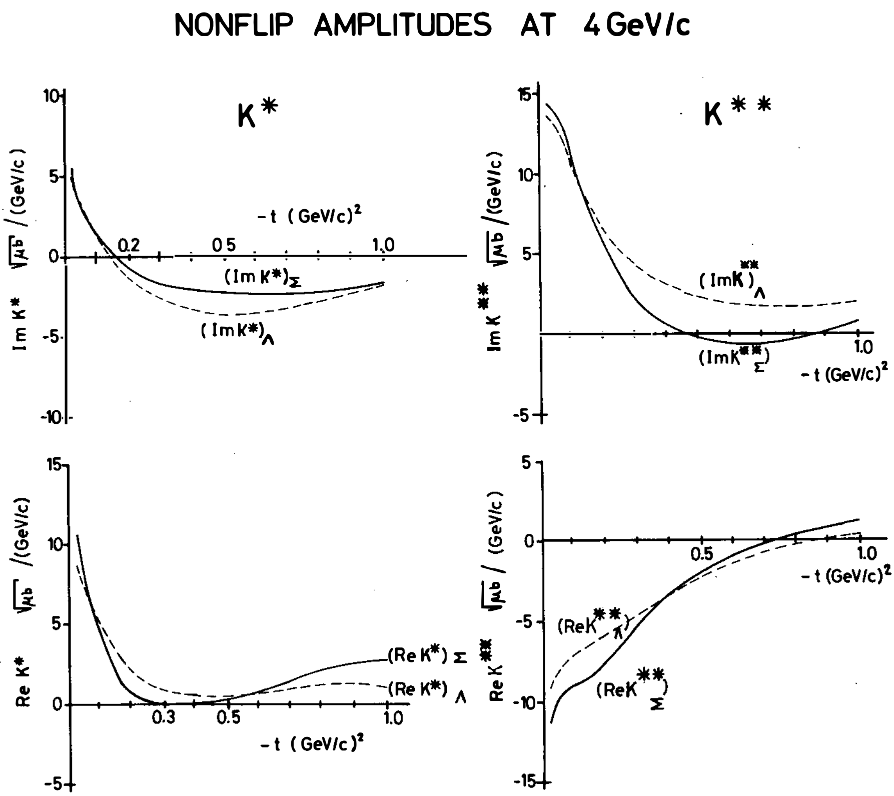

Figure 5 The K^{*} and K^{**} s channel helicity non-flip amplitudes for hypercharge exchange reactions at 4 GeV/c.

CONCLUSIONS

The over-all empirical picture which is emerging for vector and tensor exchanges in spin 0 - spin $\frac{1}{2}$ scattering is that the s channel helicity flip and also the real parts of the non-flip amplitudes show Regge pole like behaviour, whereas the imaginary parts of the non-flip amplitudes are modified. $\mathrm{Im}\, V_{++}$ shows a structure similar to that expected from a simple geometrical absorption approach while $\mathrm{Im}\, T_{++}$ cannot be so described. The various absorption prescriptions are unable to give an adequate description of the data around 5 GeV/c.

To gain insight into these puzzling features it is important to measure a complete set of observables (with accurate relative normalization of the cross-sections) at higher energies, say 10 and 20 GeV/c. This would enable the energy dependence to be studied in a region away from possible s channel resonance effects.

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